Fundamentals of a Centrifugal Fluidized Bed

A fundamental theory based on the local momentum balances is proposed to explore the fluidizing phenomena of a centrifugal particle bed. Unlike the conventional vertical bed, a centrifugal bed is predicted to be fluidized layer by layer from the inner free surface outward, in a range of aeration rates. The span of this range increases with the depth of the particle bed. The pressure drop is predicted to exhibit a plateau, which agrees very well with the observations of many investigators, but disagrees with the investigations of Takahashi et al. and Fan et al., in which the pressure drop exhibited a maximum. Predictions of the critical fluidizing velocity and the corresponding pressure drop agree well with the data of Fan et al.

Ye-Mon Chen
Department of Chemical Engineering
National Taiwan Institute of Technology
Taipei, Taiwan 107, Republic of China

Introduction

For many purposes, a fluidized bed has the merits of high heat and mass transfer rates, temperature homogeneity, and high flowability of particles. These features are particularly important for continuously operating, large-scale gas-solid reacting systems, and hence have wide applications in many industrial processes, such as FCC, fluid coking, and others. In most of these processes, vertical fluidized beds are used.

In a vertical particle bed, when the aeration is increased to a point such that the gravitational force on the particles is counterbalanced by the fluid drag exerted by the interstitial gas flow, the particle bed is fluidized. However, if a large excess amount of aeration is introduced, it simply forms large bubbles, or slugs, that pass the bed very rapidly. In such a case, the gas-solid contact becomes rather poor, due to extensive bypassing. In some operations, such as coal combustion and drying, in order to reduce the reactor size it is preferred to have high superficial gas velocity. In such processes the vertical bed becomes handicapped, so a relatively new concept, the centrifugal fluidized bed, is introduced for this special purpose.

A centrifugal fluidized bed is a cylindrical bucket rotating about its axis of symmetry with aeration introduced in the inward direction of the radius to fluidize the particles. Instead of having a fixed gravitational field as in a vertical bed, the body force in a centrifugal bed becomes an adjustable parameter that is determined by the rotation speed and the bucket radius. By using a strong centrifugal field much greater than gravity, the particle bed is able to withstand a large amount of aeration without serious formation of large bubbles and the gas-solid contact at a high aeration rate is improved.

The current address of Ye-Mon Chen is Department of Chemical Engineering, Ohio State University, Columbus, OH 43210.

Gel'perin et al. (1960, 1964) conducted some experiments on the fluidizing condition of a centrifugal bed. He correlated the Reynolds number at the minimum fluidizing condition as a unique function of the modified Galileo number. Farkas et al. (1969) proposed that the pressure drop across a centrifugal fluidized bed could be predicted by the overall balance of the centrifugal force and the drag force of the fluid. His method was found to overestimate the pressure drop. Brown et al. (1972) and Hanni et al. (1976) suggested a modified design of using a cross-flow configuration to reduce the pressure drop. A review of earlier studies on the centrifugal fluidized bed was summarized by Fan (1978).

More recently, Demircan et al. (1978) proposed a modification of the Gel'perin correlation by using Wen and Yu's (1966) correlation. Levy et al. (1978) and Kroger et al. (1979) proposed a different approach based on the local force balance at the distributor. Their theory is limited to shallow beds only. Fan et al. (1985) extended the theory to a general form for a deep bed. They suggested that the overall force balance should be used instead, and the curvature effect of the cylinder and gas inertia term were taken into account also. Their theory reduces to the model of Levy et al. (1978) if the bed is shallow enough.

In the literature, two different types of pressure drop vs. gas superficial velocity were reported. One exhibits a plateau (Metcalfe and Howard, 1977), the other exhibits a maximum instead (Fan et al., 1985), as shown in Figure 1. It is essential to justify whether the pressure drop should go through a maximum or approach a plateau on some fundamental basis. In addition, there has been some confusion in the literature concerning the definition of the minimum fluidizing condition of a centrifugal bed. In a vertical bed, it is well known that the pressure drop vs. superficial velocity exhibits a hysteresis loop in the region of the fixed bed and reaches a plateau in the region of the fluidized

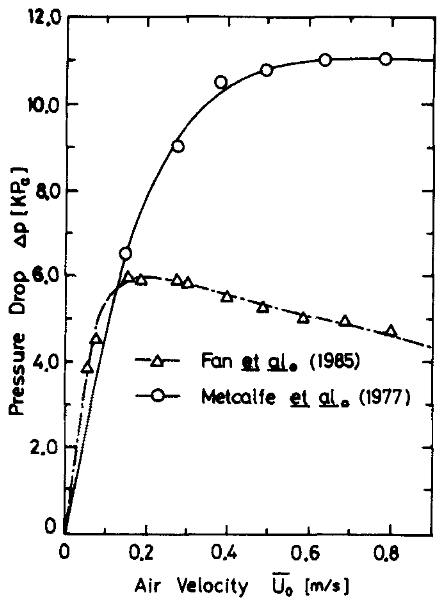

Figure 1. Two types of pressure drop vs. gas superficial velocity curves, a plateau or a maximum.

bed. The minimum fluidizing condition is defined as the point of intersection of the extrapolations from the two regions. In all experiments with a centrifugal bed, no matter whether the pressure drop was observed to exhibit a plateau or a maximum, no hysteresis loops were reported. This clearly indicates that these two types of fluidized beds have some fundamental differences. Nevertheless, the definition of the intersection of the extrapolations as the minimum fluidization has been adopted for centrifugal beds in many studies (e.g., Demircan et al., 1978; Takahashi et al., 1984). On the other hand, the minimum fluidization was taken to be the point of the maximum pressure drop in Fan et al.'s (1985) analysis. The difference between these two definitions has never been clarified.

The present study proposes a fundamental theory, based on the local momentum balances, to explore the fluidizing phenomena of a centrifugal bed. Unlike other theories, the voidage of the particle bed is allowed to vary in the present analysis so that the transition from a packed bed to a fluidized bed can be clearly followed and the fluidizing condition can then be well defined.

The present theory shows that a centrifugal bed is gradually fluidized layer by layer, from the inner surface outward, in a range of aeration rates. The aeration corresponding to initial fluidization on the inner surface is called the surface fluidizing velocity, and the aeration corresponding to complete fluidization is called the critical fluidizing velocity. The span of this aeration range, the difference between the two fluidizing velocities, increases with the depth of the particle bed. This fluidizing phenomenon is very different from that of a conventional vertical bed, which is completely fluidized at one certain aeration rate of minimum fluidizing velocity.

The theory also predicts that the pressure drop should reach a plateau beyond the critical fluidizing velocity, just like the conventional vertical bed. This prediction agrees with the observations of Kroger et al. (1979) and many others but disagrees with the observations of Takahashi et al. (1984) and Fan et al. (1985). Predictions for the critical fluidizing velocity and the corresponding pressure drop agree very well with the data of Fan et al. The accuracy of the prediction is better than that of any previous model.

Analysis

The system configuration is shown in Figure 2. r_o is the radius of the distributor and r_i is the radius of the inner surface of the particle bed. Aeration is introduced inward, from r_o to r_i. In this analysis, the centrifugal force is taken to be the dominant body force and the gravitational force is assumed to be relatively small. Under this assumption, the free surface of a vertical rotating bed is also vertical and that of a horizontal rotating bed is symmetrical to its rotating axis. The assumption of negligible gravitational force will not only simplify the problem to one-dimensional form but will also make the analysis for both the horizontal and the vertical beds identical.

If the pressure drop through the particle bed is not large, the gas phase can be assumed to be incompressible. At steady state, the equation of continuity for the gas phase is

$$\frac{d}{dr}(\epsilon r u_r) = 0 \tag{1}$$

The interstitial gas velocity is assumed to be uniformly distributed over the circumferential area and Eq. 1 can be integrated to

$$r\bar{u}_r = r_o\bar{u}_o = \text{constant} \tag{2}$$

where $\bar{u}_r = u_r\epsilon$ is the local superficial gas velocity.

If the drag force exerted on the particles by the interstitial gas is assumed to take the Ergun (1952) formula, the local momentum balance for the gas phase becomes (Jackson, 1971)

$$\frac{dP}{dr} = -\Phi_1\bar{u}_r + \Phi_2\bar{u}_r^2 + \rho_g r\omega^2 - \rho_g u_r\frac{du_r}{dr} \tag{3}$$

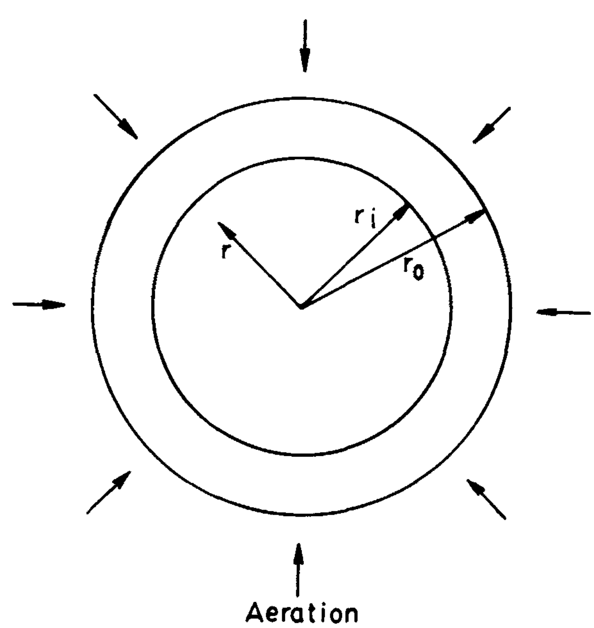

Figure 2. System configuration of a centrifugal particle bed.

Combining Eqs. 2 and 3,

$$\frac{dP}{dr} = -\Phi_1\overline{u}_r + \Phi_2\overline{u}_r^2 + \rho_g r\omega^2 + \frac{\rho_g\overline{u}_r^2}{\epsilon^2 r} + \rho_g\frac{\overline{u}_r^2}{\epsilon^3}\frac{d\epsilon}{dr} \tag{4}$$

where

$$\Phi_1 = \frac{150(1-\epsilon)^2\mu}{\epsilon^3(\phi_s d_p)^2} \quad \text{and} \quad \Phi_2 = \frac{1.75(1-\epsilon)\rho_g}{\epsilon^3\phi_s d_p} \tag{5}$$

In general, the Ergun formula, the first and second terms on the righthand side of Eq. 4, is written in terms of the slip velocity of the two phases. However, since the superficial solid velocity is zero, only the gas velocity appears in the formula.

Equation 4 is somewhat different from Fan et al.'s momentum balance. The fourth term differs by a factor of $1/\epsilon^2$. This is because the true gas velocity in the inertia term was mistaken as the superficial velocity in their analysis. The fifth term is the extra due to variable voidage. Nevertheless, the major contribution to the pressure gradient is due to the drag force, the first and second terms; the differences resulting from the fourth and fifth terms are only marginal.

The major difference between the present momentum balance and those in the literature is that the voidage is treated as a variable. Although this assumption results in additional numerical complexity of the problem, it is essential in obtaining a good estimation of the overall pressure drop. The first and second drag coefficients, Φ_1 and Φ_2, are both very sensitive functions of voidage, and a slight change in voidage will result in a considerable change in pressure drop.

Since the superficial solid velocity is zero, the local momentum balance for the solid phase is (Jackson, 1971)

$$\frac{1}{r}\frac{d}{dr}(r\sigma_r) = -\frac{dP}{dr} + (1-\epsilon)\rho_s r\omega^2 + \epsilon\rho_g r\omega^2 + \rho_g\frac{\overline{u}_r^2}{\epsilon}\left(\frac{1}{r} + \frac{1}{\epsilon}\frac{d\epsilon}{dr}\right) \tag{6}$$

Combining Eqs. 4 and 6,

$$\frac{1}{r}\frac{d}{dr}(r\sigma_r) = \Phi_1\overline{u}_r - \Phi_2\overline{u}_r^2 + (1-\epsilon)(\rho_s - \rho_g)r\omega^2 - (1-\epsilon)\rho_g\frac{\overline{u}_r^2}{\epsilon^2}\left(\frac{1}{r} + \frac{1}{\epsilon}\frac{d\epsilon}{dr}\right) \tag{7}$$

where σ_r is the normal stress in the radial direction and is taken to be positive in a compressible sense. In Eq. 7 the gravitational term is also ignored.

Fluidizing criteria

In the literature, no matter whether a criterion of local force balance (Levy et al., 1978; Kroger et al., 1979) or overall force balance (Fan et al., 1985) is applied, it has been universally assumed that at a certain gas velocity the whole centrifugal bed suddenly becomes fluidized. This gas velocity is called the incipient, or minimum fluidizing velocity. This is a well-known phenomenon in the conventional vertical bed; however, when applied to a centrifugal bed it should be investigated very carefully.

In the present analysis the criterion of fluidization is defined in a more fundamental sense. When the fluid drag of the interstitial gas is large enough to balance other forces, the normal stress will vanish. When the fluid drag exceeds other forces, a tensile stress will be acting on the solid phase. If the particles are assumed to be cohesionless, the solid phase cannot withstand any tensile stress and must be fluidized. Thus the criterion of fluidization is defined as the vanishing of the local normal stress of the solid phase.

This criterion is first tested with a conventional vertical bed. In a vertical bed, the continuity equation for the gas phase and the momentum balances for the two phases become

$$\overline{u}_z = \text{constant} \tag{8}$$

$$\frac{dP}{dz} = -\Phi_1\overline{u}_z + \Phi_2\overline{u}_z^2 + \rho_g g \tag{9}$$

$$\frac{d\sigma_z}{dz} = \Phi_1\overline{u}_z - \Phi_2\overline{u}_z^2 + (1-\epsilon)(\rho_s - \rho_g)g \tag{10}$$

where z is the coordinate downward. Since the superficial gas velocity is a constant at all levels, the fluid drag is also a constant at a given gas velocity. When the gas superficial velocity increases to the point that the fluid drag is balanced by the gravitational force, the normal stress at all levels vanishes at the same time. According to the present criterion, the whole vertical bed is indeed suddenly fluidized at one superficial velocity.

In a centrifugal bed, the gas superficial velocity is a function of position r. Therefore, the fluid drag, centrifugal force, and gas inertia are all functions of position r, and the three forces cannot balance one another at all levels at one aeration rate. In Eq. 7, the stress gradient is primarily determined by the fluid drag and the centrifugal force. Since the superficial gas velocity decreases with respect to r, the fluid drag is a monotonously decreasing function of r. On the other hand, the centrifugal force is monotonously increasing with respect to r. If the aeration rate increases from zero, the first occasion to have a zero stress gradient in the particle bed occurs on the inner surface when the superficial velocity reaches a value such that

$$-\Phi_{10}\frac{\overline{u}_{os}r_o}{r_i} + \Phi_{20}\left(\frac{\overline{u}_{os}r_o}{r_i}\right)^2 + \rho_g(1-\epsilon_0)\frac{\overline{u}_{os}^2 r_o^2}{\epsilon_0^2 r_i^3} = (1-\epsilon_0)(\rho_s - \rho_g)r_i\omega^2 \tag{11}$$

and

$$\left.\frac{d(r\sigma_r)}{dr}\right|_{r=r_i} = 0 \tag{12}$$

where

$$\Phi_{10} = \Phi_1(\epsilon_0), \quad \Phi_{20} = \Phi_2(\epsilon_0). \tag{13}$$

ϵ_0 is the bed voidage at the packed condition. Since the free surface of the particle bed has zero normal stress and Eq. 11 indicates that the stress gradient is also zero on the inner surface, the normal stress of the particle layer just beneath the inner surface vanishes at this aeration rate. According to the present criterion, the particle layer is said to be fluidized. We shall call this superficial velocity the surface fluidizing velocity, $\bar{u}_{os}$. For aeration rates smaller than this velocity, the stress gradient in the particle bed is positive at all levels and the bed remains packed.

As aeration further increases beyond the surface fluidizing velocity, the point of zero normal stress in the particle bed moves outward. According to the criterion, the particle bed is fluidized layer by layer, from the inner surface outward, as the superficial velocity increases. For a given aeration rate, the position at which the bed is fluidized, r_{pf}, can be determined by

$$-\Phi_{10}\frac{\bar{u}_o r_o}{r_{pf}} + \Phi_{20}\left(\frac{\bar{u}_o r_o}{r_{pf}}\right)^2 + \rho_g(1-\epsilon_0)\frac{\bar{u}_o^2 r_o^2}{\epsilon_0^2 r_{pf}^3} = (1-\epsilon_0)(\rho_s-\rho_g) r_{pf}\,\omega^2 \qquad (14)$$

and

$$\left.\frac{d(r\sigma_r)}{dr}\right|_{r=r_{pf}} = 0 \qquad (15)$$

When the aeration rate reaches the value such that

$$-\Phi_{10}\bar{u}_{oc} + \Phi_{20}\bar{u}_{oc}^2 + \rho_g(1-\epsilon_0)\frac{\bar{u}_{oc}^2}{\epsilon_0^2 r_o} = (1-\epsilon_0)(\rho_s-\rho_g) r_o\omega^2 \qquad (16)$$

and

$$\left.\frac{d(r\sigma_r)}{dr}\right|_{r=r_o} = 0 \qquad (17)$$

the point of zero normal stress moves to the position of the distributor and the whole particle bed is completely fluidized. This superficial velocity, corresponding to the incipient or minimum fluidizing velocity in the literature, will be called the critical fluidizing velocity, $\bar{u}_{oc}$. At this aeration rate the particles near the inner surface are already extensively fluidized and the term "incipient fluidization" could be very misleading. If the inertia and the centrifugal terms of the gas phase are negligible, Eq. 16 reduces to the criterion proposed by Levy et al. (1978).

From the preceding analysis, a centrifugal bed is not fluidized at one particular superficial velocity, but is gradually fluidized in a range of superficial velocity. The span of this range, the difference between $\bar{u}_{os}$ and $\bar{u}_{oc}$, depends mainly on the depth of the particle bed. For an extremely shallow bed, if $r_i = r_o$ the surface fluidizing velocity and the critical fluidizing velocity become identical according to Eqs. 11 and 16. Therefore, the conventional concept of fluidization at one particular gas velocity is still valid for a shallow centrifugal bed.

Voidage and pressure drop

For a given aeration rate, the pressure drop through the particle bed can easily be obtained by integrating Eq. 4 if only the bed voidage is known. In all previous models, the bed voidage was assumed to be that of the packed bed, $\epsilon = \epsilon_0$, to estimated the critical fluidizing velocity and the corresponding pressure drop. This assumption is unquestionable for an aeration rate smaller than the surface fluidizing velocity, but becomes doubtful for aeration rates larger than that.

From the preceding analysis, the particle layer is in different degrees of fluidization at different levels. Therefore, the voidage in a fluidizing region should be regarded as a variable and be determined by the local momentum balance of the solid phase. Since the stress gradient vanishes everywhere in a fluidized region, the voidage at a given aeration rate at the location of r is determined by

$$-\Phi_1\frac{\bar{u}_o r_o}{r} + \Phi_2\left(\frac{\bar{u}_o r_o}{r}\right)^2 + \rho_g(1-\epsilon)\frac{\bar{u}_o^2 r_o^2}{\epsilon^2 r^2}\left(\frac{1}{r} + \frac{1}{\epsilon}\frac{d\epsilon}{dr}\right) = (1-\epsilon)(\rho_s-\rho_g) r\omega^2 \qquad (18)$$

For the region that is not fluidized, the voidage is taken to be a constant, $\epsilon = \epsilon_0$.

Since the voidage is a variable, the particle bed is allowed to expand for aeration greater than the surface fluidizing velocity and the inner radius of the particle bed thus becomes a variable also. At a given superficial velocity, the inner radius can be determined by the overall mass balance of the particles, after the voidage is determined by Eq. 18,

$$M_s = \int_{r_i}^{r_o} \rho_s(1-\epsilon)2\pi r L\, dr \qquad (19)$$

where M_s is the total mass of particles in the bed.

The system configurations at different stages of aeration are shown in Figure 3. The pressure drops across the bed for the corresponding aeration stages are determined as follows:

Stage I, $\bar{u}_o < \bar{u}_{os}$. Since the whole particle bed remains packed, the voidage is a constant and the pressure drop through the bed is integrated to

$$\Delta P = -\Phi_{10} r_o \bar{u}_o \ln\frac{r_o}{r_i} + \Phi_{20} r_o^2 \bar{u}_o^2\left(\frac{1}{r_i} - \frac{1}{r_o}\right) + \frac{1}{2}\rho_g\omega^2(r_o^2 - r_i^2) + \frac{\rho_g r_o^2 \bar{u}_o^2}{2\epsilon_0^2}\left(\frac{1}{r_i^2} - \frac{1}{r_o^2}\right) \qquad (20)$$

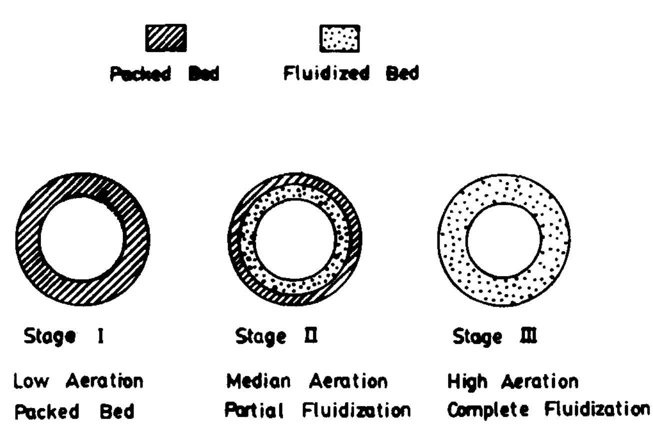

Figure 3. System configurations at different stages of aeration rate.

Stage II, $\bar{u}_{os} \le \bar{u}_o \le \bar{u}_{oc}$. In this aeration stage the particle bed is only partially fluidized. The interface of the packed bed and the fluidized bed, r_{pf}, is determined by Eq. 14, and the pressure drop is integrated to

$$\begin{aligned}\Delta P = &- \Phi_{10} r_o \bar{u}_o \ln \frac{r_o}{r_{pf}} \\ &+ \Phi_{20} r_o^2 \bar{u}_o^2 \left(\frac{1}{r_{pf}} - \frac{1}{r_o}\right) \\ &+ \frac{1}{2} \rho_g \omega^2 (r_o^2 - r_{pf}^2) \\ &+ \frac{\rho_g r_o^2 \bar{u}_o^2}{2\epsilon_0^2} \left(\frac{1}{r_{pf}^2} - \frac{1}{r_o^2}\right) \\ &+ \int_{r_i}^{r_{pf}} \left[\rho_s (1 - \epsilon) r \omega^2 \right. \\ &\left. + \epsilon \rho_g r \omega^2 + \rho_g \frac{r_o^2 \bar{u}_o^2}{\epsilon r^3}\right] dr \\ &+ \int_{r_i}^{r_{pf}} \rho_g \frac{r_o^2 \bar{u}_o^2}{\epsilon^2 r^2} \frac{d\epsilon}{dr} dr \end{aligned} \tag{21}$$

The voidage at different levels of the fluidized bed is determined by Eq. 18 and the inner radius by Eq. 19.

Stage III, $\bar{u}_{oc} < \bar{u}_o$. In this aeration stage the whole particle bed is fluidized and the pressure drop is in the form

$$\Delta P = \int_{r_i}^{r_o} \rho_s (1 - \epsilon) r \omega^2 \, dr + \rho_g \int_{r_i}^{r_o} \left(\epsilon r \omega^2 + \frac{r_o^2 \bar{u}_o^2}{\epsilon r^3} + \frac{r_o^2 \bar{u}_o^2}{\epsilon^2 r^2} \frac{d\epsilon}{dr}\right) dr \tag{22}$$

The voidage is again determined by Eq. 18 and the inner radius by Eq. 19.

It is interesting to note that Eq. 22 is under the constraint of Eq. 19. The integration of Eq. 19, the total mass of the particles, is a constant and the first integration term on the righthand side of Eq. 22 is also a constant. Since the first integration term in Eq. 22 is much larger than the second term, the pressure drop is almost a constant in stage III. This indicates that the pressure drop vs. aeration rate exhibits a plateau after the critical fluidizing velocity.

Results and Comparison

Theoretical predictions for the pressure drop vs. superficial gas velocity, are shown in Figure 4 alongside Kroger et al.'s (1979) experimental data. The three theoretical lines correspond to three different rotation speeds of the centrifugal bed. Physical data of the particles and other conditions of the Kroger et al. experiment are listed in Table 1. In their experiment, a slightly tilted, vertical rotating bed was used, with the upper distributor radius, 15.25 cm, slightly larger than the lower radius, 14.28 cm. In Table 1, the average value of the two is taken as the distributor radius. The reason for choosing the data of Kroger et al. for comparison is simply that are the most detailed and extensive experimental data for the pressure drop curves that can be found.

Theoretical predictions of the pressure drop across the particle bed are calculated by Eqs. 20 to 22, depending on the different stages of aeration. In stage I the pressure drop is found to increase almost linearly with increasing superficial velocity. In stage II the pressure drop changes smoothly from an almost linear dependence, declining to a plateau as the superficial velocity increases. In stage III the pressure drop is essentially unchanged. In Figure 4 the experimental data show a very similar tendency, as predicted. Notice in particular that the positions at which the pressure drop curves start to turn, corresponding to the surface fluidizing velocities, and the plateaus the curves finally reach, corresponding to the critical fluidizing velocities and pressure drops, agree reasonably well with the experimental data.

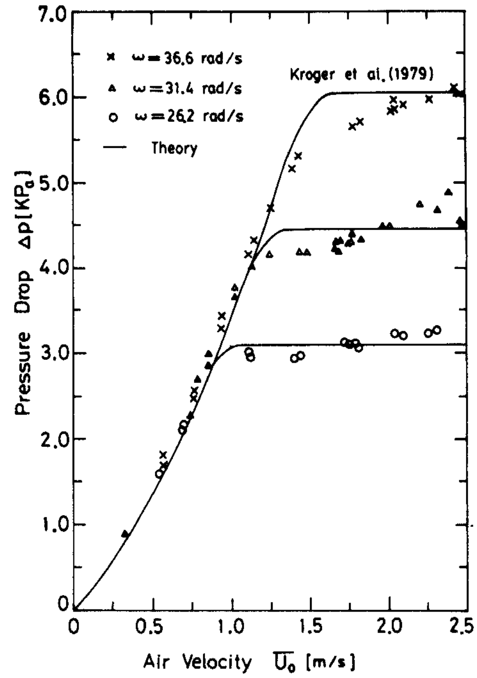

Figure 4. Comparison of predicted pressure drop vs. superficial velocity curves with experimental data of Kroger et al. (1979).

The existence of such a pressure drop plateau in a centrifugal bed was not only observed by Kroger et al. (1979), it was also reported by many other investigators (Metcalfe et al., 1977; Demircan et al., 1978; Levy et al., 1978). However, Fan et al. (1985) and Takahashi et al. (1984) reported somewhat different results, with the pressure drop exhibiting a maximum instead. Takahashi et al. claimed that their analysis could verify the existence of the maximum. However, in their analysis the bed voidage was assumed to be that of a packed bed and the analysis should be confined to the aeration rates before fluidization. On the other hand, it takes a general theory valid for aeration far

Table 1. Physical Data of Particles and Other Conditions of Kroger et al. (1979) Experiment

M_s, kg	4.54	d_p, m	0.44×10^{-3}
ρ_s, kg/m^3	2466	ρ_g, kg/m^3	1.2928
ϕ_s	1.0	ϵ_0	0.39
μ_g, kg/m · s	0.18×10^{-4}	r_o, m	0.14765
L, m	0.16		

beyond the critical fluidization to verify the existence of such a maximum. Thus, the existence of the maximum pressure drop was not verified by their analysis at all.

In Metcalfe et al.'s experiment, it was also observed that the free surface of the centrifugal bed showed bubbling taking place at aeration rates that were insufficiently large to support the bed fully. These results support the present predictions that in a centrifugal bed, fluidization indeed occurs before the critical fluidizing velocity, and fluidization begins on the free surface of the particle bed.

Fan et al. compared, quantitatively, their predictions of the critical fluidizing velocity and the corresponding pressure drop with the predictions of Levy et al. and Demircan et al., and found that their model was superior to the previous ones. Therefore, only Fan et al.'s predictions and data are quoted for quantitative comparison to verify the superiority of the present predictions.

In Figure 5, 95 experimental data of different material and different operating conditions for the critical fluidizing velocity are found to be well predicted by the present model. Most of the predictions are within 20% error, shown by the two broken lines in the figure, with an average prediction error of 9.67%, compared to an average error of 21.1% in the model of Fan et al. In Figure 6, the data of the pressure drop at critical fluidization are also found to be well predicted within 20% error, with an average error of 11.3%, compared to an average error of 23.9% for the Fan et al. model. The relative errors for the predictions are reduced by more than half by the present model.

It should be noted that in both the present model and that of Fan et al., the pressure drop at the critical fluidizing velocity tends to be overestimated, as most of the data points appear above the diagonal in Figure 6. The overestimation of the pressure drop could result from ignoring the gravitational effect by the present theory. In a vertical rotating bed, if the gravitational effect is included the free surface of the bed will not be vertical but becomes parabolic (Levy et al., 1978). Similarly, in a horizontal bed, the free surface will become asymmetric to the rotating axis, with a shallower bed above and a deeper bed below. In either case, the gravity effect generates a nonuniform bed depth. The introduced air tends to percolate through the shallower part of the bed, which might result in an overestimation of the pressure drop.

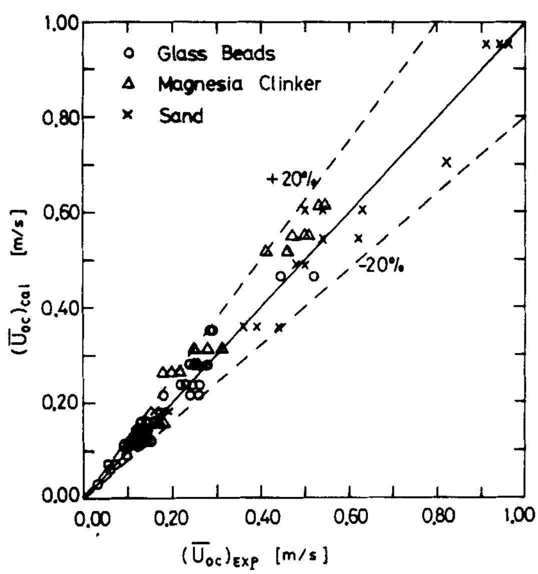

Figure 5. Comparison of predicted critical fluidizing velocities with experimental data of Fan et al. (1985).

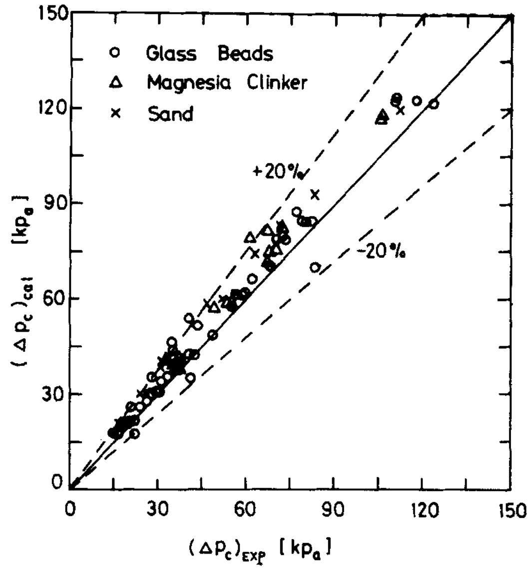

Figure 6. Comparison of predicted pressure drops at critical fluidizing velocities with experimental data of Fan et al. (1985).

Notation

d_p = mean particle diameter, m
g = gravitational acceleration, m/s^2
L = length of centrifugal bed, m
M_s = total mass of particles, kg
P = gas pressure, N/m^2
ΔP = pressure drop through particle bed, N/m^2
r = radial coordinate, m
r_{pf} = radial position of packed-fluidized interface, m
u = gas velocity, m/s
z = downward coordinate, m

Greek letters

ϵ, ϵ_0 = voidage, value at packed condition
μ = viscosity of gas phase, kg/m · s
ρ_g = density of gas phase, kg/m^3
ρ_s = density of particles, kg/m^3
ϕ_s = sphericity of particles
Φ_1, Φ_{10} = first drag coefficient, Eq. 5; value at packed condition, kg/m^3 · s
Φ_2, Φ_{20} = second drag coefficient, Eq. 5; value at packed condition, kg/m^4
ω = angular velocity of the centrifugal bed, rad/s

Subscripts and superscript

c = critical fluidization
i = at radial position of inner radius, r_i
o = at radial position of outer radius r_o
r = radial direction
s = surface fluidization
z = z direction
$\overline{\ }$ = superficial velocity

Literature cited

Brown, G. E., D. F. Farkas, and E. S. DeMarchena, "Centrifugal Fluidized Bed," *Food Technol.*, **26**, 23 (1972).

Demircan, N., B. H. Gibbs, J. Swithenbank, and D. S. Taylor, "Rotating Fluidized Bed Combustor," *Fluidization*, J. F. Davidson and D. L. Keairns, eds., Cambridge Univ. Press, 270 (1978).

Ergun, S., "Fluid Flow through Packed Column," *Chem. Eng. Prog.*, **48**, 89 (1952).

Fan, L. T., "Fluidized Bed Combustion in Centrifugal Field," *Energy Commun.*, **4**, 509 (1978).

Fan, L. T., C. C. Chang, Y. S. Yu, T. Takahashi, and Z. Tanaka, "Incipient Fluidization Condition for a Centrifugal Fluidized Bed," *AIChE J.*, **31**, 999 (1985).

Farkas, D. F., M. E. Lazar, and T. A. Butterworth, "The Centrifugal Fluidized Bed," *Food Technol.*, **23**, 125 (1969).

Gel'perin, N.J., V. G. Ainstein, and A. V. Zaikovskii, "Fluidization of Granular Material in a Centrifugal Field," *Knim. Neft. Mash.*, **3**, 1 (1960).

Gel'perin, N. J., V. G. Ainstein, and I. D. Gakman, "Rate of Inception of Fluidization and Expansion of a Fluidized Layer in a Centrifugal Force Field," *Knim. Neft. Mash.*, **5**, 18 (1964).

Hanni, P. F., D. F. Farkas, and G. E. Brown, "Design and Operating Parameters for a Continuous Centrifugal Fluidized Bed Drier," *J. Food Sci.*, **41**, 1172 (1976).

Jackson, R., "Fluid Mechanical Theory," *Fluidization*, J. F. Davidson and D. Harrison, eds., Academic Press, London, 65 (1971).

Kroger, D. G., G. Abdelnour, E. K. Levy, and J. Chen, "Flow Characteristics in Packed and Fluidized Rotating Beds," *Powder Technol.*, **24**, 9 (1979).

Levy, E., N. Martin, and J. Chen, "Minimum Fluidization and Start-up of a Centrifugal Fluidized Bed," *Fluidization*, D. L. Kearins, ed., Cambridge Univ. Press, 71 (1978).

Metcalfe, C. I., and J. R. Howard, "Fluidisation and Gas Combustion in a Rotating Fluidised Bed," *Appl. Energy*, **3**, 65 (1977).

Takahashi, T., Z. Tanaka, A. Itoshima, and L. T. Fan, "Performance of a Rotating Fluidized Bed," *J. Chem. Eng. Japan*, **17**, 333 (1984).

Wen, C. Y., and Y. H. Yu, "A Generalized Method for Predicting the Minimum Fluidization Velocity," *Chem. Eng. Symp. Ser.*, **67**, 100 (1966).

Manuscript received Apr. 23, 1986, and revision received Sept. 8, 1986.